# Design of SAW Bandpass Filters Using Weighted Least Squares (WLS) Technique

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***Abstract*–It has been demonstrated by several authors that the well-known weighted least squares (WLS) approximation can be equiripple if a suitable weighting function is applied. In the present paper, the WLS algorithm is generalized to SAW filter synthesis with prescribed magnitude and phase specifications. Several weighting techniques producing quasi-equiripple designs are presented. The frequency sampling technique is used for SAW filter frequency response approximation to reduce the number of the optimized variables. The WLS algorithm rapidly converges both for linear and non-linear phase SAW filters. Typically, no more than 5-10 iterations are required to obtain the WLS solution to accuracy better than 0.5-1 dB in the stopband when compared with the optimum Chebyshev approximation. Moreover, it is shown that the WLS technique can be effectively applied for second-order effect compensation.**

## I. INTRODUCTION

The design of SAW filters which are optimum in the Chebyshev (minimax) sense requires the use of sophisticated optimization tools such as the Remez exchange algorithm or linear programming [1, 2]. Contrary to this, for a given least squares weighting function, the optimum WLS solution can be derived analytically. The WLS software can be implemented in compact computer codes and it is included in many standard software packages. However, the standard WLS solution suffers from the drawback that the approximation accuracy deteriorates considerably near the band edges.

It has been reported by several authors [3-6] that the WLS technique produces an equiripple solution if a suitable non-uniform least squares weighting function is applied. Though there is no proof of the Chebyshev optimality for equiripple WLS designs, it has been observed that WLS optimization results closely correspond to Chebyshev optimum ones.

The major problem is that the least squares weighting function producing an equiripple design cannot be specified analytically. Therefore, iterative reweighting schemes need to be applied, with the weighting function updated at each iteration using the results of the previous iteration.

In the present paper, the WLS method is generalized to SAW filters. Several reweighting schemes are briefly discussed. The real WLS approach is extended to complex-valued (non-linear phase) SAW filter design that allows compensation for second-order effects (e.g. frequency response distortion due to electrical source/load effects).

## II. CHEBYSHEV (MINIMAX) APPROXIMATION

Consider a problem of the best approximation of the prescribed complex-valued function (target function) $D(\omega)$ by the approximating function (frequency response)

$$F(\omega) = C(\omega) + jS(\omega) = \sum_{k=0}^{N-1} a_k C_k(\omega) + j\sum_{k=0}^{N-1} b_k S_k(\omega) \quad (1)$$

where $C_k(\omega)$ and $S_k(\omega)$ are the real and imaginary part basis functions and $a_k$, $b_k$ are the approximation coefficients for the real and imaginary parts, respectively. At any frequency $\omega$, the approximation accuracy is characterized by the weighted Chebyshev error function

$$E(\omega) = W(\omega)\left[F(\omega) - D(\omega)\right] \quad (2)$$

where $W(\omega)>0$ is the Chebyshev weighting function. The Chebyshev (minimax) approximation is the best fit to the complex-valued function $D(\omega)$ to minimize an absolute error

$$\delta = \min_{\mathbf{a},\mathbf{b}} \left\| E(\omega) \right\| = \min_{\mathbf{a},\mathbf{b}} \{ \max_{\omega\in\Omega} \left| E(\omega) \right| \} \quad (3)$$

over a set of the coefficients $\mathbf{a}=[a_k]$ and $\mathbf{b}=[b_k]$ within the approximation interval $\Omega$. In the particular case of the real-valued functions (linear phase design), the key property of the optimum solution is the equiripple sign-alternated behavior of the Chebyshev error function given by the alternation theorem [1].

## III. COMPLEX WLS (CWLS) FIT

Given the complex-valued approximating function $F(\omega)$ and the desired (target) function $D(\omega)$, the WLS error on the discrete frequency grid $\omega_i\in\Omega$, $i$=0, $M$-1 is

$$\varepsilon = \sum_{i=0}^{M-1} w_i \left| F_i - D_i \right|^2 = \sum_{i=0}^{M-1} w_i \left| \sum_{k=0}^{N-1} a_k C_{ik} + j\sum_{k=0}^{N-1} b_k S_{ik} - D_i \right|^2 \quad (4)$$

where $F_i=F(\omega_i)$, $D_i=D(\omega_i)$, $C_{ik}=C_k(\omega_i)$, $S_{ik}=S_k(\omega_i)$, and $w_i$ are the WLS weights, or, in matrix form,

$$\varepsilon = (\mathbf{FA} - \mathbf{D})^* \mathbf{W} (\mathbf{FA} - \mathbf{D}) \quad (5)$$

where $\mathbf{F} = \left[\mathbf{C} \quad j\mathbf{S}\right]$, $\mathbf{A} = \left[\mathbf{a} \quad \mathbf{b}\right]^T$,

$$\mathbf{C}=\begin{bmatrix} C_{0,0} & C_{0,1} & \dots & C_{0,N-1} \\ C_{1,0} & C_{1,1} & \dots & C_{1,N-1} \\ \vdots & & \ddots & \vdots \\ C_{M-1,0} & C_{M-1,1} & & C_{M-1,N-1} \end{bmatrix}, \mathbf{S}=\begin{bmatrix} S_{0,0} & S_{0,1} & \dots & S_{0,N-1} \\ S_{1,0} & S_{1,1} & \dots & S_{1,N-1} \\ \vdots & & \ddots & \vdots \\ S_{M-1,0} & S_{M-1,1} & & S_{M-1,N-1} \end{bmatrix},$$

$$\mathbf{a}=\begin{bmatrix} a_0 & a_1 & \dots & a_{N-1} \end{bmatrix}^T, \quad \mathbf{b}=\begin{bmatrix} b_0 & b_1 & \dots & b_{N-1} \end{bmatrix}^T,$$

$$\mathbf{D}=\begin{bmatrix} D_0 & D_1 & \dots & D_{M-1} \end{bmatrix}, \mathbf{W}=diag\begin{bmatrix} w_0 & w_1 & \dots & w_{M-1} \end{bmatrix},$$

* denotes Hermitian conjugation (complex conjugation with transpose).

For the functions $D(\omega)$ and $F(\omega)$ specified on the discrete frequency grid $\omega_i\in\Omega$, $i=0, M$-1, the complex WLS (CWLS) problem is reduced to minimizing the absolute CWLS error (5).

The closed-form CWLS solution can be found by differentiating (5) with respect to the vector of the coefficients **A** and by equating the derivative to zero

$$\frac{d\varepsilon}{d\mathbf{A}}=\mathrm{Re}\left\{\mathbf{F}^{*}\mathbf{W}(\mathbf{FA}-\mathbf{D})\right\}=0 \tag{6}$$

The solution of (6) is given by

$$\mathbf{A}=\mathrm{Re}\left\{\mathbf{F}^{*}\mathbf{W}\,\mathbf{F}\right\}^{-1}\mathrm{Re}\left\{\mathbf{F}^{*}\mathbf{W}\,\mathbf{D}\right\} \tag{7}$$

By splitting (7) into the real and imaginary parts and substituting $\mathbf{A}=[\mathbf{a}\ \mathbf{b}]^{\mathrm{T}}$ we obtain

$$\begin{aligned} \mathbf{a}&=(\mathbf{C}^{\mathrm{T}}\mathbf{W}\,\mathbf{C})^{-1}\mathbf{C}^{\mathrm{T}}\mathbf{W}\,\mathrm{Re}\left\{\mathbf{D}\right\} \\ \mathbf{b}&=(\mathbf{S}^{\mathrm{T}}\mathbf{W}\,\mathbf{S})^{-1}\,\mathbf{S}^{\mathrm{T}}\mathbf{W}\,\mathrm{Im}\left\{\mathbf{D}\right\} \end{aligned} \tag{8}$$

Therefore, the CWLS problem is reduced to two separate real-valued WLS problems for the real and imaginary parts of $D(\omega)$ weighted by the same WLS weighting function $w(\omega)$. By other words, one needs to solve twice the real WLS problem for the real and imaginary parts to find the CWLS solution (8). In the particular case of the linear phase design, the WLS problem is reduced to either the real or imaginary part approximation (8).

The complexity of (8) is defined by the matrix inversion and matrix multiplications. Linear algebra techniques and commercial software can be applied to find each of the real WLS solutions (8).

## IV. CHEBYSHEV AND WLS SOLUTIONS

For comparison, the optimum Chebyshev approximation ($W_{PB}/W_{SB}$=10) and the WLS fit ($w_{SB}/w_{PB}$=8) are shown in Fig. 1 where $f_\pi=v/2p$ is the synchronous frequency, $v$ is the SAW velocity, $p$ is the transducer period. Please note that the upper case weights $W$ are related to Chebyshev approximation, whereas the lower case weights $w$ are attributed to the WLS fit.

The Chebyshev approximation is superior to the WLS near the band edges, whereas the WLS fit provides the better accuracy elsewhere. This demonstrates that by the appropriate guess for the WLS passband/stopband weight ratio the difference between WLS and Chebyshev solutions can be reduced. Moreover, by adjusting individually the WLS weights $w_i$ at each frequency point, in particular, by increasing the weights near the band edges, the overall approximation error can be minimized at the expense of the "over-approximated" regions.

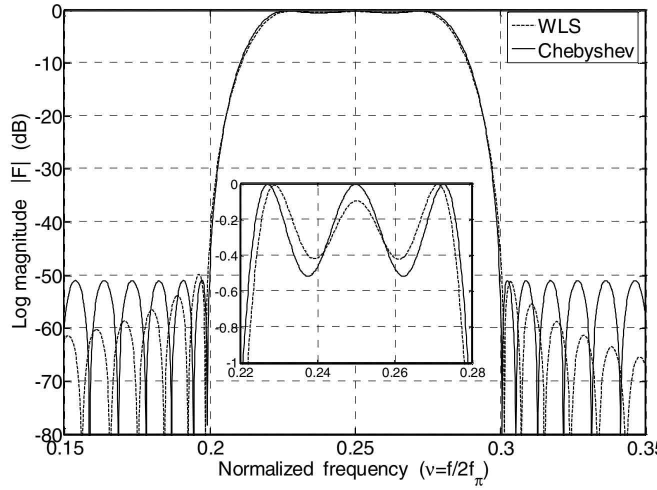


Fig. 1. Comparison of the Chebyshev ($W_{PB}/W_{SB}$=10) and WLS ($w_{SB}/w_{PB}$=8) approximations

## V. ITERATIVE REWEIGHTING SCHEMES

### *A. Principle of Reweighting*

There is no analytical method for deriving the WLS weighting function $w(\omega)$ which would produce a minimax design. An iterative approach needs to be applied, with the function $w(\omega)$ redefined at each $k$-th iteration in the following multiplicative form [4]

$$w_{k+1}(\omega)=\xi_k^{\theta}(\omega)w_k(\omega) \tag{9}$$

where $\xi_k(\omega)$>0 is the weight correction (update) function and $\theta$ is the empirical convergence factor. The normalized function $\xi_k(\omega)$ must be updated at each iteration so that $\xi_k(\omega)$>1 at frequencies where the error $|E_k(\omega)|$ needs to be reduced. At the next iteration, the error will decrease in this region at the expense of increasing in other regions. There are several reweighting schemes leading to the optimum quasi-equiripple Chebyshev solution [4-6].

### *B. Lawson's Algorithm*

The weight correction function is taken to be proportional to the weighted Chebyshev error function at the $k$-th iteration [3] (Fig. 2)

$$\xi_k(\omega)=\left|E_k(\omega)\right|=W(\omega)\left|F_k(\omega)-D(\omega)\right| \tag{10}$$

Lawson's algorithm just requires calculation of the error function value $E_k(\omega)$ at each frequency.

### *C. Step-Wise Error Approximation*

In addition to slow convergence [4], Lawson's algorithm may fail at points where $E_k(\omega)$=0, since these points give zero WLS weights ($w_{k+1}(\omega)$=0) at all further iterations that might cause divergence. Therefore, more sophisticated reweighting schemes based on the search of the error function extremal frequencies have been suggested in [4, 5]. In particular, the error function can be step-wise approximated as [5]

$$\xi_k(\omega)=S_k(\omega)=\max_{\omega_i^0\le\omega\le\omega_{i+1}^0}\left\{\left|E_k(\omega)\right|\right\}=\left|E_k(\omega_i)\right|, \tag{11}$$

where $\omega_i$ is the position of the local maximum (extremal frequency), and $\omega_i^0$, $\omega_{i+1}^0$ are the positions of the local minima (valley frequencies) of the absolute Chebyshev error function $|E_k(\omega)|$ (Fig. 2).

*D. Error Envelope Approximation*

The best convergence is obtained using a reweighting scheme based on the following approximation [4]:

$$\xi_k(\omega) = R_k(\omega) \tag{12}$$

where $R_k(\omega)$ is the envelope of the absolute error function $|E_k(\omega)|$ reconstructed as a set of line segments connecting the extremal frequencies in a particular frequency subband (Fig. 2).

The two last weighting schemes involve searching for extremal frequencies, with special care taken in the extrema interpretation at the band edges [4]. However, the simplest algorithms for extrema search can be applied, since the iterative WLS method does not require high accuracy in the location and evaluation of extrema.

The ultimate optimum WLS solution does not depend on the reweighting scheme while the convergence speed and hence the number of iterations are rather sensitive to reweighting.

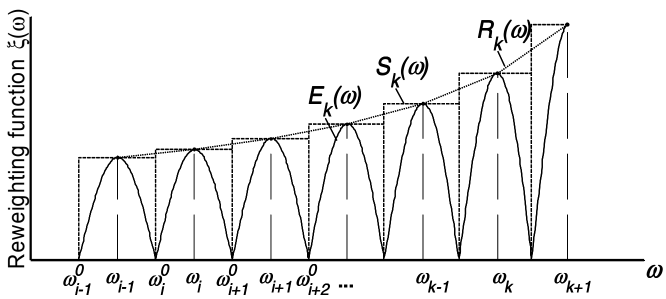


Fig. 2. Different reweighting functions: $E_k(\omega)$ is the Chebyshev error function, $S_k(\omega)$ is the step-wise error approximation, $R_k(\omega)$ is the error function envelope

*E. Convergence*

The number of WLS iterations needed to closely approximate the optimum Chebyshev solution depends on the exponential convergence factor $\theta$. Again, there is no analytical technique for obtaining $\theta$ that provides the best algorithm convergence. From our experience, good convergence has been observed for $\theta \approx 0.5$-$0.75$.

The iterative WLS algorithm can be terminated after the prescribed number of iterations or after checking the equiripple behavior to a required accuracy. Typically, the WLS algorithm converges within 5-10 iterations to accuracy better than 0.5-1 dB in the stopband when compared with the optimum Chebyshev approximation. In our example, it has taken 15 Lawson's iterations to obtain an equiripple Chebyshev approximation shown in Fig. 1 with accuracy better than 0.5 dB. About twice as less iterations based on the step-wise or envelope error function approximations are needed to achieve the same accuracy. The uniform WLS weights $w_i$=1 have been used as the initial guess and the value of the convergence factor has been chosen as $\theta$ =0.5. The ultimate normalized LWS weights in the passband and stopband are shown in Fig. 3 for all three reweighting schemes giving the same Chebyshev solution in Fig. 1.

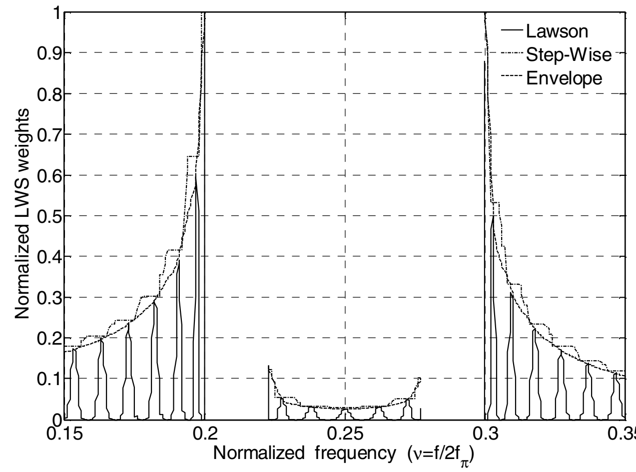


Fig. 3. Normalized WLS weights for different reweighting schemes

The WLS computational time is comparable with the linear programming [6], while the latter is much more sophisticated in programming and cumbersome for practical use.

## VI. SAW FILTER DESIGN

*A. CWLS Algorithm Modifications*

The iterative WLS technique can be applied to SAW filter synthesis with linear and non-linear phase responses. In general case, the input SAW transducer response and the element factor should be accounted for in the passband by modifying the weighting function $W(\omega)$ and target function $D(\omega)$ [7].

To reduce the WLS problem size (and hence the computational time), the frequency sampling technique [1, 2] can be applied

$$\begin{aligned} C(\varphi) &= \frac{1}{N}\sum_i C(\varphi_i)\{\mathrm{sinc}(\varphi-\varphi_i)+\mathrm{sinc}(\varphi+\varphi_i)\} \\ S(\varphi) &= \frac{1}{N}\sum_i S(\varphi_i)\{\mathrm{sinc}(\varphi-\varphi_i)-\mathrm{sinc}(\varphi+\varphi_i)\} \end{aligned} \tag{13}$$

where $\mathrm{sinc}(\varphi)=\dfrac{\sin N\varphi/2}{\sin\varphi/2}$, $\varphi=\beta p=\pi\dfrac{f}{f_\pi}$, $\beta=\omega/v$ is the SAW wave number and $N$ is the number of transducer fingers. The second term in (13) accounts for the contribution to the frequency response of the "mirror" band (with respect to the synchronous frequency $f_\pi$) which is in-phase (+ sign) with the baseband response for the real part $C(\varphi)$ and in anti-phase (– sign) for the imaginary part $S(\varphi)$. For bandpass SAW filters, most of the frequency samples at the frequency points $\varphi_i=i\Delta\varphi$, $\Delta\varphi=2\pi/N$ can be set to zero values without sacrificing significantly the approximation accuracy.

*B. Second Order Effect Compensation*

The iterative CWLS algorithm can be applied to compensation of some second order effects, in particular, electrical source/load effects. The SAW filter transfer function is given by

$$S_{12} = \frac{Y_{12}Y_L}{Y_{12}Y_{21} - (Y_{11}+Y_0)(Y_{22}+Y_L)} \approx \frac{-Y_{12}Y_L}{(Y_{11}+Y_0)(Y_{22}+Y_L)} \quad (14)$$

where $Y_{ik}$, $i,k$=1,2 are $Y$-parameters and $Y_{0,L}$ are the source and load admittances, respectively. Matching circuits can be also accounted for in (14).

In the quasi-static approximation, it is the SAW filter trans-admittance $Y_{12}(\omega)=Y_{21}(\omega)$ that depends on the linear function of the transducer tap weights or frequency samples. As $S_{12}$ depends on all $Y$-parameters, the synthesis problem is non-linear and an iterative compensation procedure is needed, with the target function predistorted in correspondence with the simulation results from the previous iteration. Therefore, we are approaching a “moving target” using as reference the “ideal” transfer function $S_{12}^{(0)}$ at each compensation iteration, so that

$$D^{(i)} = D^{(i-1)} \frac{S_{12}^{(0)}}{S_{12}^{(i-1)}}, \quad D^{(0)} = D, \quad S_{12}^{(0)} = Y_{12}^{(0)} / Y_0. \quad (15)$$

At each compensation iteration, the CWLS problem is constructed and solved by using (8), (9).

### *C. Design Example*

A matched SAW filter frequency response is shown in Fig. 4 before and after compensation of the electrical source/load effects (50 Ω system). Two series inductors $L_1$=240 $n$H and $L_2$=160 $n$H ($Q$=30) have been used for matching. The substrate material is $LiTaO_3$, the acoustic aperture is W=2.2 mm (67 $\lambda$). The number of transducer fingers is $N_1$=100 (withdrawal-weighted) and $N_2$=700 (apodized), respectively. The number of the optimized frequency samples is 50. The discrete frequency grid comprises 500 points in the frequency range 60-80 MHz.

The number of iterations for the electrical circuit effects compensation was 5, while the number of the WLS iterations at each compensation step was 10. The error function envelope technique has been applied for reweighting. There is almost no residual magnitude and phase distortion in the passband after the compensation.

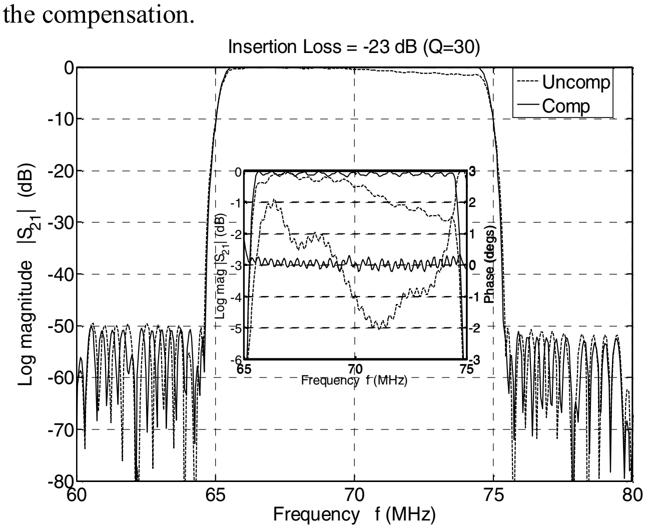


Fig. 4. SAW filter design example

## VII. CONCLUSIONS

The iterative WLS procedure has been developed for SAW filter design with arbitrary magnitude and phase specifications. Due to its simplicity and straightforward implementation, the proposed WLS approach to the Chebyshev approximation is a powerful alternative to the linear programming and the Remez exchange algorithm. The WLS technique is computationally efficient and easy for programming. Three different reweighting schemes resulting in the same Chebyshev solution have been discussed, with the only difference in the number of WLS iterations required to achieve an acceptable accuracy. Typically, the Chebyshev approximation can be achieved in just 5-10 WLS iterations to the accuracy better than 0.5-1 dB in the SAW filter stopband if compared with the Chebyshev problem solution. It has been demonstrated that the iterative WLS method can be effectively applied for compensation of second order effects. The SAW filter design examples have been presented which confirm the advantages of the iterative WLS design technique.

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